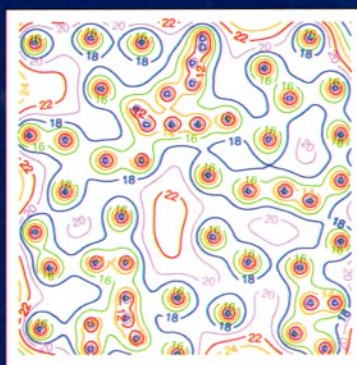
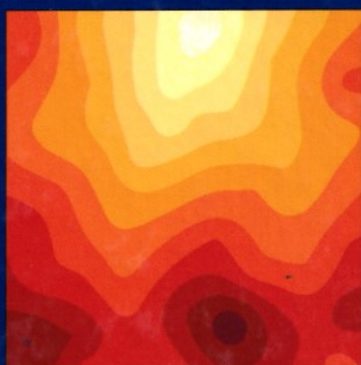
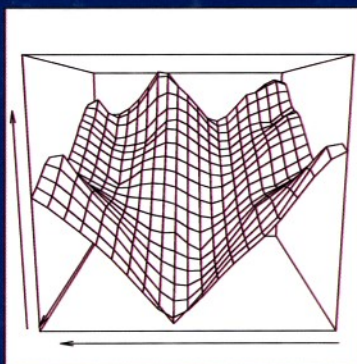


Wiley Series in Probability and Statistics

SPATIAL ANALYSIS



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Kent and Mardia are also joint authors of a well-established monograph on *Multivariate Analysis*.

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