

RIVKA GITIK | LEO JOSKOWICZ

# COMPUTATIONAL GEOMETRY

with Independent  
and Dependent Uncertainties



 World Scientific

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This comprehensive compendium describes a parametric model and algorithmic theory to represent geometric entities with dependent uncertainties between them. The theory, named Linear Parametric Geometric Uncertainty Model (LPGUM), is an expressive and computationally efficient framework that allows to systematically study geometric uncertainty and its related algorithms in computer geometry.

The self-contained monograph is of great scientific, technical, and economic importance as geometric uncertainty is ubiquitous in mechanical CAD/CAM, robotics, computer vision, wireless networks and many other fields. Geometric models, in contrast, are usually exact and do not account for these inaccuracies.

This useful reference text benefits academics, researchers, and practitioners in computer science, robotics, mechanical engineering and related fields.

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