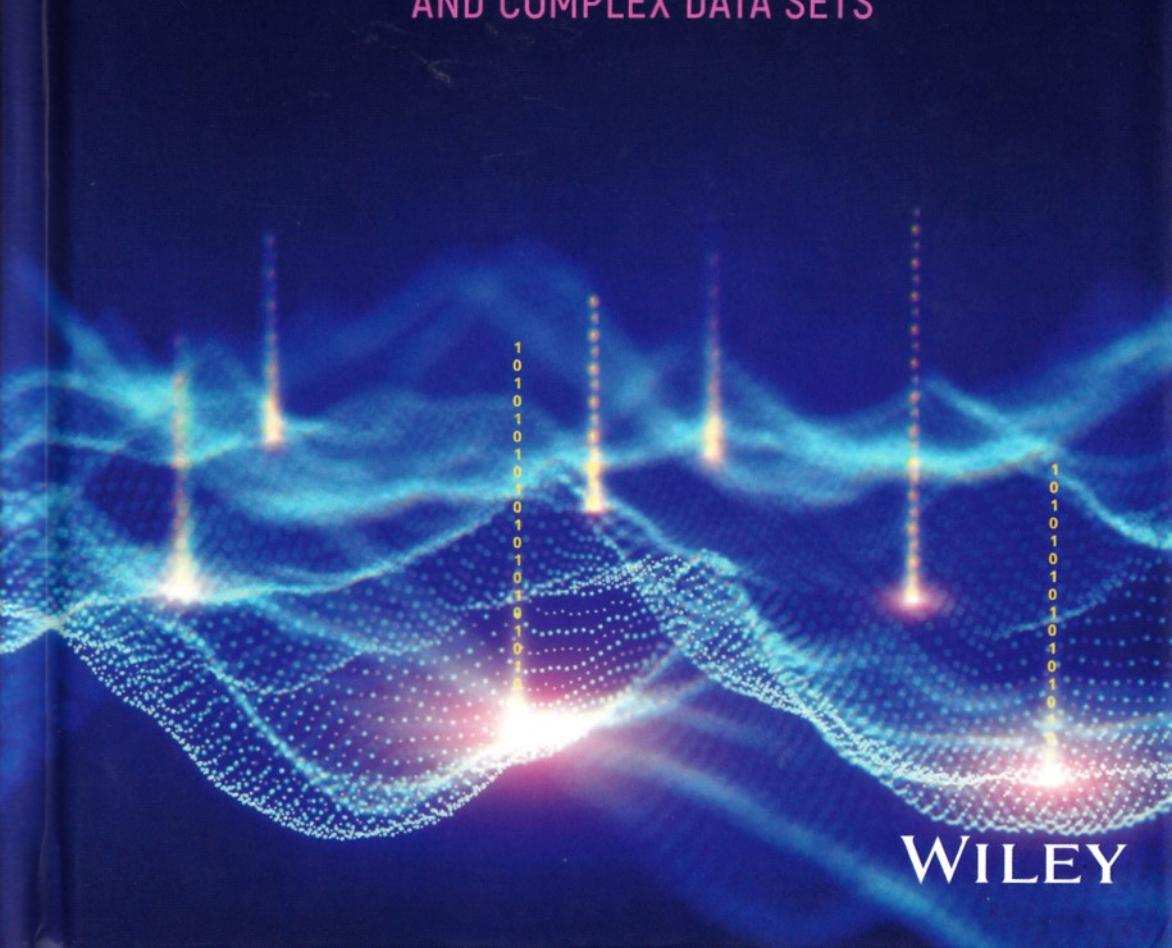


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DATA SCIENCE IN THEORY AND PRACTICE

TECHNIQUES FOR BIG DATA ANALYTICS
AND COMPLEX DATA SETS



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Data Science in Theory and Practice delivers a comprehensive treatment of the mathematical and statistical models useful for analyzing data sets arising in various disciplines, like banking, finance, health care, bioinformatics, security, education, and social services. Written in five parts, the book examines some of the most commonly used and fundamental mathematical and statistical concepts that form the basis of data science. The authors go on to analyze various data transformation techniques useful for extracting information from raw data, long memory behavior, and predictive modeling.

The book offers readers a multitude of topics all relevant to the analysis of complex data sets. Along with a robust exploration of the theory underpinning data science, it contains numerous applications to specific and practical problems. The book also provides examples of code algorithms in R and Python and provides pseudo-algorithms to port the code to any other language.

Ideal for students and practitioners without a strong background in data science, readers will also learn from topics like:

- Analyses of foundational theoretical subjects, including the history of data science, matrix algebra and random vectors, and multivariate analysis
- A comprehensive examination of time series forecasting, including the different components of time series and transformations to achieve stationarity
- Introductions to both the R and Python programming languages, including basic data types and sample manipulations for both languages
- An exploration of algorithms, including how to write one and how to perform an asymptotic analysis
- A comprehensive discussion of several techniques for analyzing and predicting complex data sets

Perfect for advanced undergraduate and graduate students in Data Science, Business Analytics, and Statistics programs, *Data Science in Theory and Practice* will also earn a place in the libraries of practicing data scientists, data and business analysts, and statisticians in the private sector, government, and academia.

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